2011 Higher Paper 2.

1 (a) $B(4,4,0)$

(b) $\overrightarrow{DB} = \underline{b} - \underline{d}$

$$= \begin{pmatrix}4\\4\\0\end{pmatrix} - \begin{pmatrix}2\\2\\6\end{pmatrix}$$

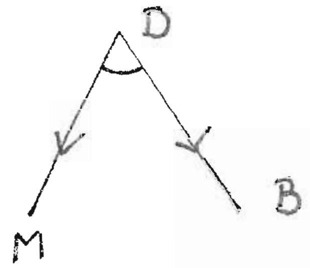
$$= \underline{\underline{\begin{pmatrix}2\\2\\-6\end{pmatrix}}}$$

$\overrightarrow{DM} = \underline{m} - \underline{d}$

$$= \begin{pmatrix}2\\0\\0\end{pmatrix} - \begin{pmatrix}2\\2\\6\end{pmatrix}$$

$$= \underline{\underline{\begin{pmatrix}0\\-2\\-6\end{pmatrix}}}$$

(c) $\cos BDM = \dfrac{\overrightarrow{DB}.\overrightarrow{DM}}{|\overrightarrow{DB}||\overrightarrow{DM}|}$



$\cos BDM = \dfrac{32}{\sqrt{44} \times \sqrt{40}}$

$\cos BDM = 0{\cdot}76277\ldots.$

$\angle BDM = \cos^{-1}(0{\cdot}76277\ldots)$

$\angle BDM = \underline{\underline{40{\cdot}3^\circ}}$

$\overrightarrow{DB}.\overrightarrow{DM} = 2\times0 + 2\times(-2) + (-6)\times(-6$

$= 0 + (-4) + 36$

$= 32$

$|\overrightarrow{DB}| = \sqrt{2^2 + 2^2 + (-6)^2} = \sqrt{44}$

$|\overrightarrow{DM}| = \sqrt{0^2 + (-2)^2 + (-6)^2} = \sqrt{40}$

2 (a) $g(f(x))$

$= g(x^3 - 1)$

$= 3(x^3 - 1) + 1$

$= \underline{\underline{3x^3 - 2}}$

(b) $g(f(x)) + x\,h(x)$

$= 3x^3 - 2 + x.(4x - 5)$

$= 3x^3 - 2 + 4x^2 - 5x$

$= \underline{\underline{3x^3 + 4x^2 - 5x - 2}}$

(c) i)

1	3	4	-5	-2
	↓	3	7	2
	3	7	2	$\boxed{0}$

Since $f(1) = 0$
$\underline{\underline{(x-1) \text{ is a factor.}}}$

c ii) $3x^3 + 4x^2 - 5x - 2$

$= (x-1)(3x^2 + 7x + 2)$

$= \underline{\underline{(x-1)(3x+1)(x+2)}}$

2(d) $g(f(x)) + xh(x) = 0$

$3x^3 + 4x^2 - 5x - 2 = 0$

$(x-1)(3x+1)(x+2) = 0$

$\underline{\underline{x = -2, -\frac{1}{3}, 1}}$

3.(a) $u_{n+1} = -\frac{1}{2}u_n \qquad u_0 = 16$

$u_1 = -\frac{1}{2}u_0$

$u_1 = -\frac{1}{2} \times 16 = -8$

(b) $V_{n+1} = pV_n + q \qquad V_1 = 4, \; V_2 = 5, \; V_3 = 7$

$V_2 = pV_1 + q$

$5 = 4p + q$

$q = 5 - 4p$

$V_3 = pV_2 + q$

$7 = 5p + q$

$q = 7 - 5p$

Hence $5 - 4p = 7 - 5p$

$\underline{\underline{p = 2}}$

when $p = 2$

$q = 5 - 4(2)$

$\underline{\underline{q = -3}}$

(c) i, $u_{n+1} = -\frac{1}{2}u_n$

A limit exists since $-1 < -\frac{1}{2} < 1$

$\text{Limit} = \frac{b}{1-a} = \frac{0}{1-(-\frac{1}{2})} = 0$

ii, $V_{n+1} = 2V_n - 3 \qquad [\, V_{n+1} = aV_n + b \,]$

For a limit to exist $-1 < a < 1$

as $2 > 1$ a limit does not exist.

4. $A_1 = \int_{-2}^{0} x^3 - x^2 - 4x + 4 - (2x+4)\, dx$

$= \int_{-2}^{0} x^3 - x^2 - 4x + 4 - 2x - 4 \quad dx$

$= \int_{-2}^{0} x^3 - x^2 - 6x \quad dx$

$= \left[\frac{x^4}{4} - \frac{x^3}{3} - \frac{6x^2}{2} \right]_{-2}^{0}$

$= \left(\frac{0^4}{4} - \frac{0^3}{3} - \frac{6(0^2)}{2} \right) - \left(\frac{(-2)^4}{4} - \frac{(-2)^3}{3} - \frac{6(-2)^2}{2} \right)$

$= (0) \qquad - \left(\frac{16}{4} - \frac{(-8)}{3} - 12 \right)$

$= -\left(4 + \frac{8}{3} - 12\right)$

$= 8 - \frac{8}{3}$

$= 5\frac{1}{3}$ unit^2

$A_2 = \int_{0}^{3} 2x + 4 - (x^3 - x^2 - 4x + 4)\, dx$

$= \int_{0}^{3} 2x + 4 - x^3 + x^2 + 4x - 4 \quad dx$

$= \int_{0}^{3} 6x + x^2 - x^3 \, dx$

$= \left[\frac{6x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \right]_{0}^{3}$

$= \left(\frac{6(3)^2}{2} + \frac{3^3}{3} - \frac{3^4}{4} \right) - \left(\frac{6(0)^2}{2} + \frac{0^3}{3} - \frac{0^4}{4} \right)$

$= \left(27 + 9 - \frac{81}{4} \right) \quad - (0)$

$= \left(36 - 20\frac{1}{4} \right) \quad - (0)$

$= 15\frac{3}{4}$ units^2

4. Total Area $= A_1 + A_2$

$$= 5\tfrac{1}{3} + 15\tfrac{3}{4}$$

$$= 20\tfrac{1}{3} + \tfrac{3}{4}$$

$$= 20\tfrac{4}{12} + \tfrac{9}{12}$$

$$= 20 + \tfrac{13}{12}$$

$$= \underline{\underline{21\tfrac{1}{12} \text{ units}^2}}$$

5. $y = kx^n$

Taking $\log_2$ of both sides

$$\log_2 y = \log_2 kx^n$$

$$\log_2 y = \log_2 k + \log_2 x^n$$

$$\log_2 y = n\log_2 x + \log_2 k$$

$$Y = mX + c$$

$$n\text{ (gradient)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-5}{4-0} = \underline{\underline{\frac{1}{2}}}$$

$(0,5)$ $(4,7)$

$$\log_2 k \text{ (y intercept)} = 5$$

$$\log_2 k = 5$$

$$k = 2^5$$

$$\underline{\underline{k = 32}}$$

RADIANS

6. $3\sin x - 5\cos x = R\sin(x + a)$

$$3\underline{\sin x} - 5\underline{\cos x} = R\underline{\sin x}\cos a + R\underline{\cos x}\sin a$$

Equating coefficients

$$R\cos a = 3$$

$$R\sin a = -5$$

$$R^2 = 3^2 + (-5)^2$$

$$R^2 = 34$$

$$R = \sqrt{34}$$

S	A✓
T	C✓

$\tan a = R\sin a$ ... -5

6 (a) $\tan a = -\frac{5}{3}$

Calc. Radian Mode $\longrightarrow$ $\tan^{-1}\left(\frac{5}{3}\right) = 1.03$ radians

(a lies in quadrant 4)

$a = 2\pi - 1.03$ radians

$\underline{a = 5.3 \text{ radians to 1 dp.}}$

Hence $3\sin x - 5\cos x = \underline{\sqrt{34}\sin(x + 5.3)}$

(b) $\int_0^t 3\cos x + 5\sin x \, dx = 3$

$\left[3\sin x - 5\cos x \right]_0^t = 3$

$\left[\sqrt{34}\sin(x + 5.3) \right]_0^t = 3$

$\sqrt{34}\sin(t + 5.3) - \left(\sqrt{34}\sin(5.3)\right) = 3$

$\sqrt{34}\sin(t + 5.3) - (-4.9) = 3$

$\sqrt{34}\sin(t + 5.3) + 4.9 = 3$

$\sqrt{34}\sin(t + 5.3) = -1.9$

$\sin(t + 5.3) = \frac{-1.9}{\sqrt{34}}$

S	A
T ✓	C

$\sin^{-1}\left(\frac{1.9}{\sqrt{34}}\right) = 0.3$

Use radian mode on your calc.

$t + 5.3 = (\pi + 0.3), (2\pi - 0.3)$

$t + 5.3 = 3.4, 6.0$

$t = (3.4 - 5.3), (6.0 - 5.3$

$t = -1.9, 0.7$

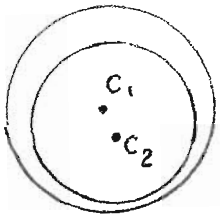
but $0 < t < 2$

Equivalent angle

$-1.9 + 2\pi = 4.4$ ra

$t = 0.7$ radians to 1 Dp.

7.



The circles do not touch.

Circle C_1 $(x+1)^2 + (y-1)^2 = 121$

Centre C_1 $(-1, 1)$

$\text{Radius}_1 = \sqrt{121} = 11$.

Circle C_2 $x^2 + y^2 - 4x + 6y + \quad = 0$

$$2g = -4 \qquad 2f = 6$$
$$g = -2 \qquad f = 3$$

Centre $(-g, -f)$

Centre C_2 $(2, -3)$

$$\text{Radius}_2 = \sqrt{g^2 + f^2 - c}$$
$$= \sqrt{(-2)^2 + 3^2 - p}$$
$$= \sqrt{13 - p}$$

For circle to exist

$$\text{Radius} > 0$$
$$\text{Radius}^2 > 0$$
$$13 - p > 0$$
$$-p > -13$$
$$p < 13$$

Distance between the centres

$$|C_1C_2| = \sqrt{(2-(-1))^2 + (-3-1)^2}$$
$$= \sqrt{3^2 + (-4)^2}$$
$$= \sqrt{25}$$
$$= 5$$

As the circles donot touch

$$\text{Radius}_2 < 11 - 5$$
$$\text{Radius}_2 < 6$$
$$\sqrt{13 - p} < 6$$
$$13 - p < 36$$
$$-p < 23$$
$$p > -23$$

Range $-23 < p < 13$