SQA Higher Maths 2011 Paper 1 SOLUTIONS

① $2\underset{\sim}{p} - \underset{\sim}{q} - \frac{1}{2}\underset{\sim}{r} = 2\begin{pmatrix}2\\5\\-7\end{pmatrix} - \begin{pmatrix}1\\0\\-1\end{pmatrix} - \frac{1}{2}\begin{pmatrix}-4\\2\\0\end{pmatrix} = \begin{pmatrix}4\\10\\-14\end{pmatrix} - \begin{pmatrix}1\\0\\-1\end{pmatrix} - \begin{pmatrix}-2\\1\\0\end{pmatrix} = \begin{pmatrix}5\\9\\-13\end{pmatrix}$ C

② $\left.\begin{aligned} 3y + 2x &= 6 \\ 3y &= -2x + 6 \\ y &= -\tfrac{2}{3}x + 2 \end{aligned}\right\} m = -\tfrac{2}{3}$ B

③ $y = f(x+2) - 1$ is horizontal shift 2 units to left then vertical down one. D

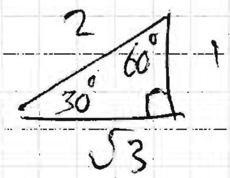
④ $y = x^3 - 2x$. Gradient comes from $\frac{dy}{dx}$ D
$\frac{dy}{dx} = 3x^2 - 2$. When $x = 2$, $\frac{dy}{dx} = 10$

⑤ $x^2 - 8x + 7 = (x^2 - 8x) + 7 = (x^2 - 8x + 16) - 16 + 7$
$= (x-4)^2 - 9$ A

⑥ $m_{radius} \times m_{tangent} = -1$. So $m_{tangent} = \frac{1}{2}$
Eq. of tangent is $y - (-3) = \frac{1}{2}(x - 2)$
$y + 3 = \frac{1}{2}(x-2)$ C

⑦ Use 1
$$\begin{array}{c|cccc} 1 & 1 & -1 & 1 & 3 \\ & & 1 & 0 & 1 \\ \hline & 1 & 0 & 1 & \underline{4} \end{array}$$
Remainder = D

⑧ $m = \tan\theta$. So $m = \tan 30^\circ = \frac{1}{\sqrt{3}}$ A



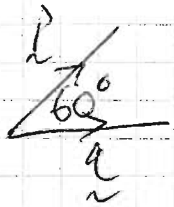
⑨ Discriminant ≥ 0, so Roots are real
Discriminant $\neq$ perfect square (e.g. 4, 9, 16, 25)
So roots not rational B

⑩ $2\cos x = \sqrt{3}$ see triangle in ⑧
$\cos x = \frac{\sqrt{3}}{2}$
$x = 30^\circ$ or $(360^\circ - 30)$
$= \frac{\pi}{6}$ or $\frac{11\pi}{6}$ D

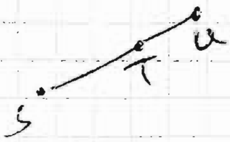
⑪ $\int (4x^{\frac{1}{2}} + x^{-3})\,dx = \dfrac{4x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \dfrac{x^{-3+1}}{-3+1} + c = \dfrac{4x^{3/2}}{\frac{3}{2}} - \dfrac{x^{-2}}{2} + C$ D
$= \frac{8}{3}x^{3/2} - \frac{x^{-2}}{2} + c$

⑫ $\left.\begin{aligned} \sin p &= \tfrac{o}{h} = \tfrac{2}{\sqrt{5}}, & \sin q &= \tfrac{2}{3} \\ \cos p &= \tfrac{a}{h} = \tfrac{1}{\sqrt{5}}, & \cos q &= \tfrac{\sqrt{5}}{3} \end{aligned}\right\}$ $\sin(p+q)$
$= \sin p\cos q + \cos p \sin q$
$= \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{3} + \frac{1}{\sqrt{5}} \times \frac{2}{3}$
$= \frac{2}{3} + \frac{2}{3\sqrt{5}}$ C

⑬ $f(x) = 4\sin 3x$, $f'(x) = 4\cos 3x \times 3$ Chain rule
$= 12\cos 3x$
$f'(0) = 12\cos 0 = 12$ C

(14) 

$\underset{\sim}{p} \cdot \underset{\sim}{q} = |\underset{\sim}{p}||\underset{\sim}{q}|\cos 60° = 3 \times 3 \times \frac{1}{2} = \frac{9}{2}$ B

(15) 

$\overrightarrow{ST} = \underset{\sim}{t} - \underset{\sim}{s} = \begin{pmatrix}-16\\-4\\16\end{pmatrix} - \begin{pmatrix}-4\\5\\1\end{pmatrix} = \begin{pmatrix}-12\\-9\\15\end{pmatrix}$

$\overrightarrow{SU} = \underset{\sim}{u} - \underset{\sim}{s} = \begin{pmatrix}-24\\-10\\26\end{pmatrix} - \begin{pmatrix}-4\\5\\1\end{pmatrix} = \begin{pmatrix}-20\\-15\\25\end{pmatrix}$

$\overrightarrow{ST} = \frac{3}{5}\overrightarrow{SU}$ So, $\left.\begin{matrix}ST = 3\\ SU = 5\end{matrix}\right\}$ $\left.\begin{matrix}ST = 3\\ TU = 2\end{matrix}\right\}$

T divides SU in ratio 3:2 B

(16) $\int \frac{1}{3x^4}\,dx = \int \frac{1}{3}x^{-4}\,dx$ the 3 does not move!

$= \frac{1}{3}\int x^{-4}\,dx = \frac{1}{3}\frac{x^{-3}}{-3} + c = -\frac{1}{9}x^{-3} + c$ A

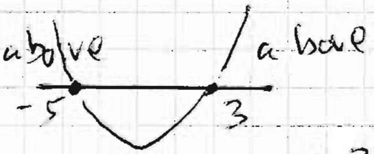
(17) Roots at $-1, 0, 2$ mean factors of $(x-(-1))(x-0)(x-2)$

$= (x+1)x(x-2)$

So either A or C Test using $(1,2)$

A: $y = -1(1+1)(1-2) = 2$ Bingo! A

(18) Draw quadratic $(x-3)(x+5)$
MIN SP since x^2 term is positive



$x < -5, \quad x > 3$ C

(19) Various methods!
e.g. A is impossible since straight lines thro' origin are $y = mx$

e.g. $\log_3 y = x$. Use inverse function 3 to the power

$3^{(\log_3 y)} = 3^x$

$y = 3^x$ $\therefore$ when $x = 0$, $y = 3^0 = 1$ $\therefore$ must be C

e.g. substitute values
$B(1,1)$ $\log_3 1 = 1$ False since $\log_3 3 = 1$
etc.

(20) For $g(x) = \sin^2\sqrt{x-2}$ means $\left(\sin\sqrt{x-2}\right)^2$

$x - 2 \geq 0$ $\longrightarrow$ No square roots of negative nos.

$x \geq 2$

So C or D

$\sin(?)$ is always ≤ 1 and ≥ -1

So $\sin(?)$ squared is always ≥ 0 and ≤ 1 D

(21) (a) $m_{BD} = \frac{15}{5} = 3$ Eq. $y - 12 = 3(x-7)$

$y = 3x - 21 + 12$

$y = 3x - 9$

(b) $BD: y = 3x - 9$
$AC: x + 3y = 23$ } So, $x + 3(3x-9) = 23$

$x + 9x - 27 - 23 = 0$

$10x = 50$

$x = 5$ So $y = 3x - 9$
$= 6$

$E(5, 6)$

(c) $m_{AB} = \frac{4}{8} = \frac{1}{2}$ $\therefore m_{\text{perp bisector}} = -2$ from $m_1 m_2 = -1$

Midpoint of $AB = \left(\frac{-1+7}{2}, \frac{8+12}{2}\right) = (3, 10)$

Eq. of perp bisector is $y - 10 = -2(x-3)$

$y = -2x + 6 + 10$

$y = -2x + 16$

With $E(5,6)$ then $y = -2 \cdot 5 + 16 = -10 + 16 = 6$ ✓

(22) (a) (i) x-axis: $y = 0$ here So $(x-2)(x^2+1) = 0$

$x - 2 = 0$ $\quad x^2 + 1 = 0$

$x = 2$ $\quad$ No solns.

y-axis: $x = 0$ here So $y = (0-2)(0^2+1) = -2$

x-axis $(2, 0)$
y-axis $(-2, 0)$

(b) $f(x) = (x-2)(x^2+1) = x^3 - 2x^2 + x - 2$

$f'(x) = 3x^2 - 4x + 1$

For S.P., $f'(x) = 0$ $\therefore 3x^2 - 4x + 1 = 0$

$(3x - 1)(x - 1) = 0$

$x = \frac{1}{3}$ or 1

x	$\rightarrow$	$\frac{1}{3}$	$\rightarrow$	1	$\rightarrow$
$f'(x)$	+	0	−	0	+
Shape	/	—	\	—	/

when $x = \frac{1}{3}$, $y = \left(\frac{1}{3} - 2\right)\left(\frac{1}{9} + 1\right)$

$= -\frac{5}{3} \cdot \frac{10}{9}$

$= -\frac{50}{27}$

when $x = 1$, $y = (1-2)(1^2+1)$

$= -2$

$f'(0) = 3(0)^2 - 4(0) + 1 = +1$

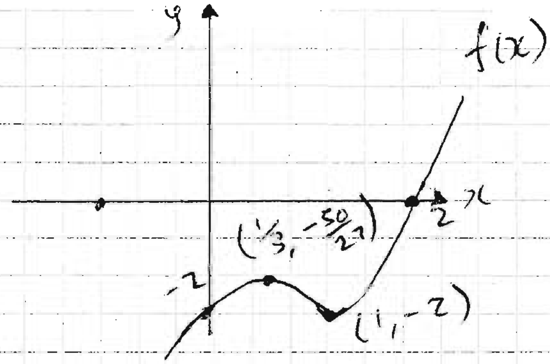
$f'(\frac{1}{2}) = 3(\frac{1}{2})^2 - 4(\frac{1}{2}) + 1 = -\frac{1}{4}$

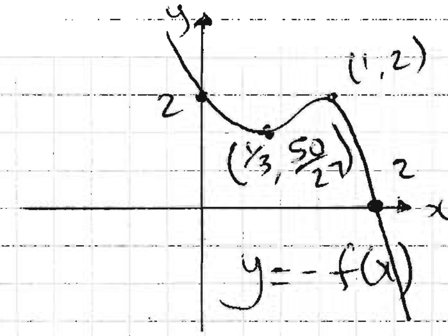
$f'(2) = 3(2)^2 - 4(2) + 1 = 5$

MAX. SP $\left(\frac{1}{3}, -\frac{50}{27}\right)$

MIN SP $(1, -2)$

(22) (c)





(23) (a) $\cos 2x^\circ - 3\cos x^\circ + 2 = 0$

Use $\cos 2x^\circ$ double angle formula $2\cos^2 x^\circ - 1$

$(2\cos^2 x^\circ - 1) - 3\cos x^\circ + 2 = 0$

$2\cos^2 x^\circ - 3\cos x^\circ + 1 = 0$

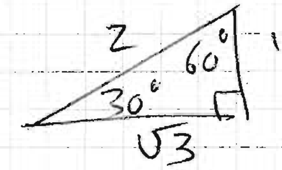
Compare with $2x^2 - 3x + 1$
$(2x-1)(x-1)$

$(2\cos x^\circ - 1)(\cos x^\circ - 1) = 0$

$\cos x^\circ = \frac{1}{2}$ or $\cos x^\circ = 1$

$x^\circ = 60^\circ$ or 300°

$x = 0^\circ$ or 360°



Since $0 \leq x < 360$, $x = 0^\circ, 30^\circ, 300^\circ$

(b) Consider $\cos 4x^\circ - 3\cos 2x^\circ + 2 = 0$

This is the same equation essentially as (a)

Think of it as $\cos(2(2x)^\circ) - 3\cos(2(2x^\circ)) + 2 = 0$

So the solutions will still be 0, 60, 300

But this time it is $2x = 0, 60, 300$

So $x = 0, 30, 150$

But you also have the solutions which are separated by the period of $\cos 2x^\circ$ which is 180°

So, full solutions are $x = 0, 30, 150, (0+180)$
$(30+180), (150+180)$

Hence, $x = 0, 30, 150, 180, 210, 330$